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Mixed Electromagnetic and Circuit Simulations using Higher-Order Elements and Bases

N. J. Champagne[*], J. D. Rockway[*], and V. Jandhyala[†]

Abstract - **In this paper, an approach to couple higher-order electromagnetic surface integral equations to circuit simulations is presented. Terminals are defined that connect circuit elements to contacts modeled on the distributed electromagnetic domain. A modified charge-current continuity equation is proposed for a generalized KCL connection at the contacts. The distributive electromagnetic integral equations are developed using higher-order bases and elements that allow both better convergence and accuracy for modeling. The resulting scheme enables simultaneous solution of electromagnetic integral equations for arbitrarily-shaped objects and SPICE-like modeling for lumped circuits, and permits design iterations and visualization of the interaction between the two domains.**

1 INTRODUCTION

Surface integral equation formulations are desirable for simulating packaging and interconnect structures due to the related ease in modeling arbitrary geometries and equivalent current flow [1]. Specifically, the Rao-Wilton-Glisson (RWG) approach [2] based on triangular surface tessellations permit modeling of arbitrarily shaped structures and arbitrarily directed equivalent surface currents. These forms of modeling are particularly useful for package and system-on-chip simulation and can also enable tightly coupled circuit and electromagnetic simulation [3].

At high frequencies, surface impedance approximations are sufficiently accurate to model losses and inductive behavior caused by skin effects. However, at lower frequencies approaching DC, cross sections of conductors are smaller than the skin depth. Standard surface impedance approximations are invalid. Therefore, for broadband simulation as necessitated in digital or ultra-wideband systems, a volumetric formulation is typically required at these low frequencies. In a volumetric formulation, the skin effect can be modeled explicitly. This modeling requires fine and frequency dependent volume meshing. It is noted that some recent efforts have been aimed at obtaining new surface impedance approximations that might be valid at low frequencies. These are typically restricted to cases of assumed or uniform cross sections [4] as opposed to more general three-dimensional structures, such as packages and on-chip inductors.

Handling a mix of full-wave and skin-like effects with a surface-only formulation is desirable since frequency-dependent effects can be tracked without changing geometric discretization and without making recourse to a special volume formulation at low frequencies. This is particularly true for small microelectronic structures where geometry detail and not wavelength is the guiding factor in mesh discretization. To accomplish a surface-only formulation valid for realistic conductors over a broad range of frequencies, the interior lossy medium EM problem must be addressed and coupled to the external medium model. This can be accomplished using the PMCHWT formulation [1,5].

The intent in this paper is to develop the formulation necessary to use higher-order basis and element representations to provide better resolution and accuracy for the modeling of integrated circuit (IC) and packaging problems. This will be described in terms of a tightly coupled circuit and PMCHWT-based electromagnetic simulation. In the next section, the integral equation formulation, as employed in EIGER [6], is briefly presented, including the higher-order basis and element representations. In section three, the coupled PMCHWT-Circuit formulation is developed. Finally, some results and conclusions of the formulation are presented in terms of circuit elements coupled to these higher-order integral equations.

2 INTEGRAL EQUATION FORMULATION

Since we are dealing with penetrable bodies in this paper, the scattered electric and magnetic fields are represented in terms of the external and internal mixed potential integral equations (MPIE) as

$$\mathbf{E}_{\pm}^{s}(\mathbf{r}) = \mp jw\mathbf{A}^{\pm}(\mathbf{r}) \mp \nabla\Phi^{\pm}(\mathbf{r}) \mp \frac{1}{e^{\pm}}\nabla\times\mathbf{F}^{\pm}(\mathbf{r}) \tag{1}$$

and

$$\mathbf{H}_{\pm}^{s}(\mathbf{r}) = \mp jw\mathbf{F}^{\pm}(\mathbf{r}) \mp \nabla\Psi^{\pm}(\mathbf{r}) \pm \frac{1}{m^{\pm}}\nabla\times\mathbf{A}^{\pm}(\mathbf{r}), \tag{2}$$

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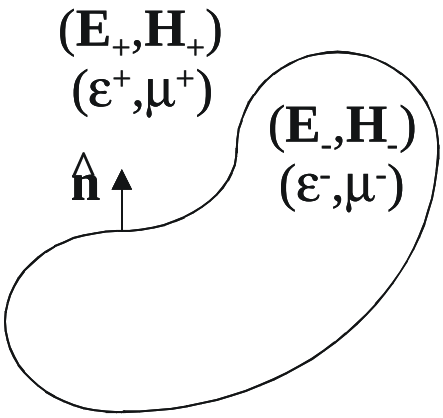

Figure 1: A penetrable body that is in space.

where $\mathbf{E}_+$ and $\mathbf{E}_-$ are the electric fields of the exterior and interior regions, respectively, as shown in Fig. 1. The magnetic fields in (2) are represented in a similar way. The potentials in (1) and (2) are given by

$$\mathbf{A}(\mathbf{r}) = m\int_S G(\mathbf{r},\mathbf{r}')\,\mathbf{J}(\mathbf{r}')\,dS', \tag{3}$$

$$\Phi(\mathbf{r}) = \frac{-1}{jwe}\int_S G(\mathbf{r},\mathbf{r}')\,\nabla'\cdot\mathbf{J}(\mathbf{r}')\,dS', \tag{4}$$

$$\mathbf{F}(\mathbf{r}) = e\int_S G(\mathbf{r},\mathbf{r}')\,\mathbf{M}(\mathbf{r}')\,dS', \tag{5}$$

and

$$\Psi(\mathbf{r}) = \frac{-1}{jwm}\int_S G(\mathbf{r},\mathbf{r}')\,\nabla'\cdot\mathbf{M}(\mathbf{r}')\,dS', \tag{6}$$

where $G(\mathbf{r},\mathbf{r}')$ is the homogeneous Green's function, $\mathbf{J}(\mathbf{r})$ and $\mathbf{M}(\mathbf{r})$ are the surface electric and magnetic currents, respectively. The electric and magnetic surface currents are expanded in a series of linearly independent basis functions as

$$\mathbf{J}(\mathbf{r}) \approx \sum_{n=1}^{N_J} J_n \mathrm{L}_n(\mathbf{r}) \tag{7}$$

and

$$\mathbf{M}(\mathbf{r}) \approx \sum_{n=1}^{N_M} M_n \mathrm{L}_n(\mathbf{r}), \tag{8}$$

where the J_ns and M_ns are the unknown surface current coefficients, N_J and N_M are the number of unknown electric and magnetic currents, respectively, and the L_ns are the basis functions.

2.1 Basis Function Representation

The surface of interest is discretized into triangular surface patch elements and/or quadrilateral surface patch elements. The basis functions on an element are represented in terms of Sylvester-Lagrange polynomials as presented in [7]. For example, the divergence conforming bases on a triangular element, such as that in Fig. 2, are written as

$$\mathrm{L}^b_{ijk}(\mathbf{r}) = N^b_{ijk}\frac{(p+2)x_b\,\hat{a}_{ijk}(x)}{i_b}\mathrm{L}_b(\mathbf{r}), \tag{9}$$

where p is the basis order, x_b is the local coordinate, $\hat{a}_{ijk}$ is the product of modified Sylvester polynomials, and L_b is the zeroth-order basis. The remaining terms are described in detail in [7]. The divergence of (9) is given by

$$\nabla\cdot\mathrm{L}^b_{ijk}(\mathbf{r}) = N^b_{ijk}\frac{(p+2)}{i_b}\left[\frac{2x_b\,\hat{a}_{ijk}(x)}{\mathrm{J}} + \nabla x_b\cdot\hat{a}_{ijk}(x)\,\mathrm{L}_b(\mathbf{r}) + x_b\nabla\hat{a}_{ijk}(x)\cdot\mathrm{L}_b(\mathbf{r})\right], \tag{10}$$

where J is the Jacobian of the element.

2.2 Element Representation

It should be noted that the element geometry may be interpolated using the Sylvester-Lagrange polynomials, too. The expression for finding a point on an element is simpler than the bases and is represented as

$$\mathbf{r} = \sum_{ijk}\mathbf{r}_{ijk}a_{ijk}(x), \tag{11}$$

where a_{ijk} is the product of the Sylvester polynomials and $\mathbf{r}_{ijk}$ is the interpolation point on an element. Also, if the mesh file format is based on the ordering of the local coordinates, then no element mappings are required for each element interpolation order (q). Hence, higher-order elements may be utilized without needing a unique mapping for each element and its particular interpolation order.

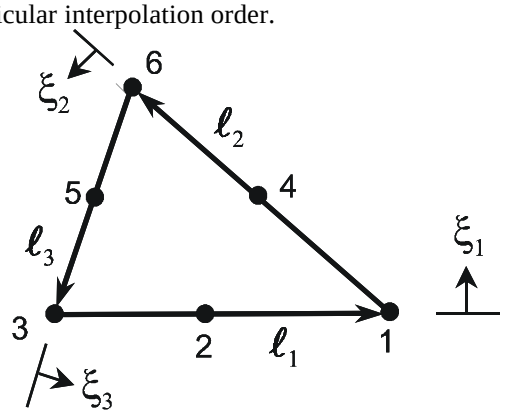

Figure 2: The quantities associated with a quadratic (q=2) triangle.

2.1.2 Circuit -Connection Algorithm

The contact-connection algorithm [1] is employed to couple the circuit to the electromagnetic surface. A circuit is attached to a spatially localized surface S_c by enforcing at this contact a modified current-continuity equation, a KCL connection, and a KVL connection from the contact to the circuit node. This is shown in Fig. 3. These three conditions are fundamental to the coupled PMCHWT-Circuit formulation. On a contact surface S_c, the continuity equation is changed to account for injecting branch current from the circuit. This current introduces an additional source term in the continuity equation and thus affects the distribution of both the electromagnetic surface currents and surface charges. Hence, the continuity equation is modified to become

$$\nabla \cdot \mathbf{J} + jwr = \begin{cases} I_c, & r \in S_c, \\ 0, \text{ otherwise}, \end{cases} \tag{12}$$

where I_c is the contact current. This contact current provides a virtual extension from the distributive electromagnetic surface to the circuit node. The coupling between the electromagnetic formulation and the circuit is done by enforcing (1) and (2) at the boundary and coupling them to the circuit by including the contact current from (12). The localized circuit source attached to the contact produces an additional source or sink of charge that alters the scalar potential and the resulting electric field. Because of this additional current, the scalar potentials must be tied to the circuit node voltage V_n. A KVL expression sets the scalar potentials at the equipotent circuit voltage V_n. Finally, the contact current is connected to the circuit by including an addition term I_c to the KCL based circuit equation associated at circuit node n.

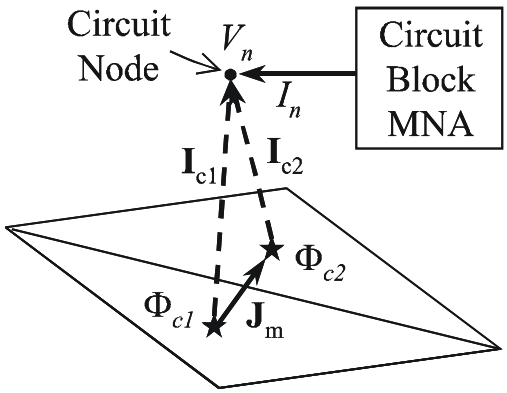

Figure 3: This is the connection scheme for the contact-circuit algorithm.

The PMCHWT-Circuit formulation, including the connecting KCL and equipotential KVL equations, may be summarized as the following block-matrix equation:

$$\begin{bmatrix} EM & X \\ -X^T & MNA \end{bmatrix} \begin{bmatrix} I_{EM} \\ I_{ckt} \end{bmatrix} = 0. \tag{13}$$

where the *EM* block matrix represents the PMCHWT equations and contacts, the X and X_T block matrices are the connection matrices between circuit nodes and contacts, the *MNA* block matrix represents the modified nodal circuit analysis, and I represents the currents.

3 RESULTS

A rectangular connector (1 mm × 1 mm × 4 mm) is modeled to determine the convergence of the resistance curve around the first resonance of the structure. The results shown in Fig. 4 are for the p = 0, 1, and 2 basis function orders. The results are essentially converged only after using the next higher-order basis (p = 1) from the p = 0 (RWG) basis function order.

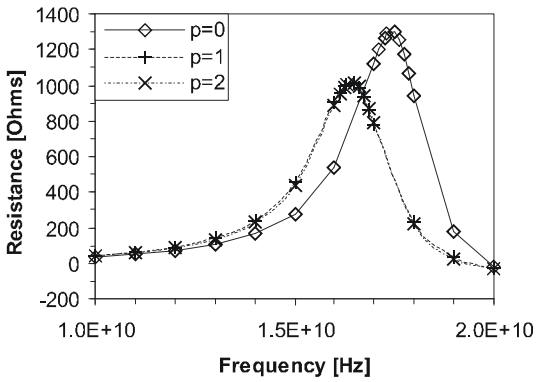

Figure 4: A plot of the resistance versus frequency for a rectangular connector.

4 CONCLUSIONS

A brief description of the PMCHWT formulation was presented along with the higher-order elements and basis functions used in the analysis. A technique for representing circuit-to-EM connections was utilized in the analysis. Results were presented to demonstrate the usefulness of using higher-order formulations. In the specific example presented, only the next order basis function beyond the RWG basis was needed for convergence to occur.

Acknowledgments

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